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Study of rogue wave by low-pass filtered noise in a multistable fiber laser

Estudio de onda gigante por ruido filtrado pasa baja en un láser de fibra multiestable

R. Jaimes-Reátegui*, R. Sevilla-Escoboza*, G. Huerta-Cuéllar*, J. H. García-López*, D. López Mancilla*, C. E. Castañeda-Hernández*, A. N. Pisarchik**

ABSTRACT

Clear evidence of rogue waves in a multistable system is revealed with an erbium-doped fiber laser driven by harmonic pump modulation (Pisarchik, Jaimes-Reátegui, Sevilla-Escoboza, Huerta-Cuellar & Taki, 2011). We demonstrate numerically and experimentally that a low-pass noise filtering can control the probability for the appearance of a particular state. The results of numerical simulations with the use of a three-level laser model display good agreement with experimental results. The mechanism for the rogue wave formation lies in the interplay of stochastic processes with multistable deterministic dynamics. Low-frequency noise applied to a diode pump current induces rare jumps to coexisting subharmonic states with high-amplitude pulses perceived as rogue waves. The probability of these events depends on the noise filtered frequency and grows up when the noise amplitude increases. The probability distribution of spike amplitudes confirms the rogue wave character of the observed phenomenon.

RESUMEN

Clara evidencia de ondas gigantes en un sistema multiestable es revelada con un láser de fibra dopada con erbium conducida por modulación de bombeo (Pisarchik, Jaimes-Reátegui, Sevilla-Escoboza, Huerta-Cuellar & Taki, 2011). Se demuestra numérica y experimentalmente que un filtro de ruido pasa baja aplicado a la modulación de bombeo del láser puede controlar la probabilidad de aparición de un particular estado de respuesta del láser de fibra. Los resultados de simulaciones numéricas con el uso de un modelo de láser de tres niveles muestran gran concordancia con los resultados experimentales. El mecanismo para la formación de ondas gigantes radica en la interacción de procesos estocásticos con dinámica determinística multiestable. Aplicación de ruido de baja frecuencia a la corriente del diodo de bombeo induce raros saltos de estados subarmónicos coexistentes con pulsos de alta amplitud percibidos como ondas gigantes. La probabilidad de estos eventos depende de la frecuencia del ruido filtrada, y crece cuando la amplitud del ruido aumenta. La distribución de probabilidad de las amplitudes de pulsos confirma el carácter de onda gigante del fenómeno observado.

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INTRODUCTION

The interaction between stochasticity and nonlinearity is a central current issue in the study of different dynamical systems, including radiophysical, climatic, populational, geophysical, epidemical, and optical models. Several experimental and theoretical works have demonstrated that this interaction sometimes plays a positive role in multistable systems, e.g., stochastic, coherence, and vibrational resonances, attractor hopping (Pisarchik, Jaimes-Reátegui, Sevilla-Escoboza, Huerta-Cuéllar & Taki, 2011; Pisarchik & Jaimes-Reátegui, 2009), noise-enhanced multi-stability, and pre-bifurcation noise amplification. Under certain circumstances, noise creates preference for some attractors in a multistable system (Pisarchik, Kir´yanov, Barmenkov & Jaimes-Reátegui, 2005; Pisarchik, Jaimes-Reátegui, Sevilla-Escoboza & Huerta-Cuéllar, 2005a).

*Centro Universitario de Los Lagos, Universidad de Guadalajara. Enrique Díaz de León, Paseos de la Montaña, Lagos de Moreno, Jalisco, México. 47460. E-mail: rjaimes@culagos.udg.mx
**Centro de Investigaciones en Óptica. Loma del Bosque 115, Lomas del Campestre, León, Guanajuato, México. 37150.

We study under what conditions rare giant pulses with short duration and large amplitude can appear in a multistable system. To do so, we selected a diode-pumped erbium-doped fiber laser (EDFL), because its coexisting attractors are well known (Pisarchik *et al.*, 2005a; Pisarchik *et al.*, 2005b; Pisarchik *et al.*, 2011; Pisarchik & Jaimes-Reátegui, 2009). This work is organized as follows: In Section 2 we introduce the fiber laser model. In section 3 we give the experimental setup. In section 4 we discuss the results, and finally in section 5 we give the main conclusion.

Theoretical model

The fiber laser is described by the balance equations for the laser intensity P and the average population in the active medium to the upper level (Pisarchik & Jaimes-Reátegui, 2009).

$$\frac{dP}{dt} = \frac{2L}{T_r} P \left\{ r_w \alpha_0 \left[N(\xi - \eta) - 1 \right] - \alpha_{th} \right\} + P_{sp}, \tag{1}$$

$$\frac{dN}{dt} = -\frac{\sigma_{12} r_w P}{\pi r_0^2} (N\xi - 1) - \frac{N}{\tau} + P_{pump}, \tag{2}$$

$$P_{sp} = N \frac{10^{-3}}{\tau T_r} \left(\frac{\lambda_g}{w_0} \right)^2 \frac{r_0^2 \alpha_0 L}{4\pi^2 \sigma_{12}}, \tag{3}$$

$$P_{pump} = P_p \frac{1 - \exp\left[-\alpha_p L (1 - N) \right]}{N_0 \pi r_0^2 L}, \tag{4}$$

$$P_p = P_p^0 [1 + m \sin(2\pi F_m t) + \kappa G(\zeta, f_n)]. \tag{5}$$

Where N is the population of the upper laser level, n_0 is the refractive index of a "cold" erbium doped fiber (EDF) core, and L is the active fiber length, σ_{12}level "1" level "2". $\sigma_{12} = \sigma_{21}$that yields $\xi = (\sigma_{12} + \sigma_{21}) / (\sigma_{12}) = 2$, $\eta = (\sigma_{23}) / (\sigma_{12}) = 0.4$at the laser wavelength. $T_r = ((2n_0) / c)(L + l_0)$,$(l_0$ being the intra-cavity tails of the fiber Bragg grating (FBG) couplers, $\alpha_0 = N_0 \sigma_{12}$ is the small-signal absorption of the erbium fiber at the laser wavelength, $N_0 = N_1 + N_2$ being the total concentration of erbium ions in the active fiber, $\alpha_{th} = \gamma_0 + (1 / 2L) ln(1 / R)$ is the intra-cavity loss on the threshold $(\gamma_0$ being the non-resonant fiber loss and R is the total reflection coefficient of the FBG couplers), τ-level "2", r_0 is the fiber core radius, w_0 is the radius of the fundamental fiber mode, $r_w = 1 - exp[-2(r_0) / (w_0)^2]$, is the factor addressing a match between the laser fundamental mode and erbium-doped core volumes inside the active fiber. In the equation (3) P_{sp} is the spontaneous emission into the fundamental laser mode. We assume here that the laser spectrum width is 10-3 of the erbium luminescence spectral bandwidth (λ_g is the laser wavelength). In the equation (4) P_{pump} is the pump power, where P_p is the pump power at the fiber entrance and $\beta = \alpha_p / \alpha_0$ is the dimensionless coefficient that accounts for the ratio of absorption coefficients of the erbium fiber at pump wavelength λ_p to that at laser wavelength λ_g. To account for the pump modulation, one needs to write the pump power at the active fiber P_p as in equation (5), where a sinusoidal character of modulation is supposed, with modulation frequency F_m and modulation depth m.

Experimental setup

The experimental setup figure 1, is similar to that described in previous papers of some of the authors (Pisarchik *et al.*, 2011; 2005b). A 1560 *nm* EDFL is pumped by a 977 *nm* diode pump laser. The 4.81*m* Fabry-Perot laser cavity is formed by an active 88 *cm* long heavily erbium-doped fiber with a 2.7 *μm* core diameter and two fiber Bragg gratings (FBGs) with 0.288 *nm* and 0.544 *nm* full widths on half magnitude bandwidth, having respectively 100% and 95.88% reflectivities at the laser wavelength. In our experiments the diode current is fixed at 145.5 *mA* corresponding to a 20 *mA* pump power, while the threshold current is 110 *mA*.

To drive the EDFL, the sum of harmonic and random modulations, $m \sin\left(2\pi F_m t\right) + \kappa G\left(\zeta, f_n\right)$, from signal and noise generators is applied to the diode pump current . Here, m and F_m are, respectively, the amplitude and frequency of the external harmonic modulation, κ is the noise amplitude, and $G(\zeta, f_n)$ is the noise function in terms of a random number $\zeta \in -1, 1]$ and the noise cutoff frequency fn (white noise is filtered with a fifth order discrete low-pass Butterworth filter in Labview 8 .5). The pump current was chosen to ensure a laser relaxation oscillation frequency around $f_r = 30$ *kHz* in absence of external noise.

Connections shown in figure 1 are as follow. First the Wave Function Generator (WFG) signal is added with the Digital to Analogical Converter Card (DAC) that send the noise signal from an interface (IF). The added signals enter as external modulation into the Laser Diode Controller (LDC) that is connected with the 980 nm Laser Diode (LD). The signal from LD enters to the 980 nm/1550 nm Wave Divison Multiplexer (WDM) and it it used to pump the Er Doped Fiber (EDF). The laser cavity is made using a pair of FBG (FBG1 and FBG2). An Optical Isolator (OI) is placed at the exit of the laser cavity followed by a Photo Detector (PD) that converts the optical signal to an electrical signal that can be seen using an Oscillator (OS).

RESULTS

Since we are interested in the parameter region where the laser exhibits multistability, we explore $F_m = 90\ KHz$ for which $P1$, $P3$, and $P4$ coexist. Figure 2 shows the experimentally observed rogue wave (figure 3 numerical result). The laser switches to $P4$ for only a few (3-5) periods and then falls back into the regular $P1$ regime. For the explored parameters, these events occur very rarely; the average time between the consequent rogue waves is about 30s (Pisarchik *et al.*, 2005a). By varying the noise parameters f_n and, one can control the switching probability for different coexisting states. To verify whether the observed spikes exhibit the rogue wave properties, we calculate the probability density functions (PDFs) from experimental time series obtained for different noise amplitudes, see figure 4 and figure 5 numerical results. The pulses with a very large intensity appear much more often than they would according to gaussian statistics; that confirms their rogue wave character. In the presence of noise, the number of coexisting attractors changes; *i.e.*, the multi stable system is converted to a metastable one (Pisarchik *et al.*, 2005a) where the peak intensity is not the same as it was in the noiseless system.

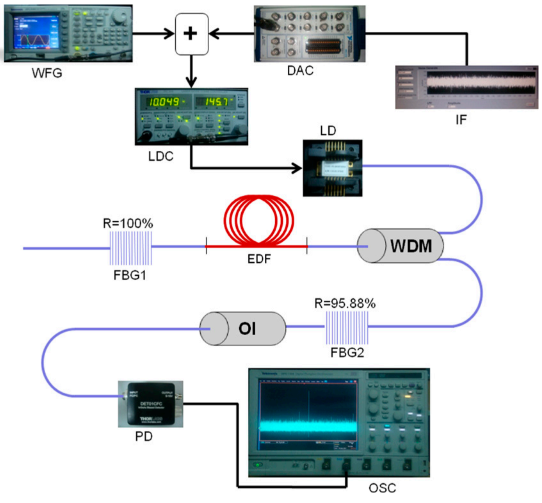

Figure 1. Experimental setup.
Source: Authors own elaboration.

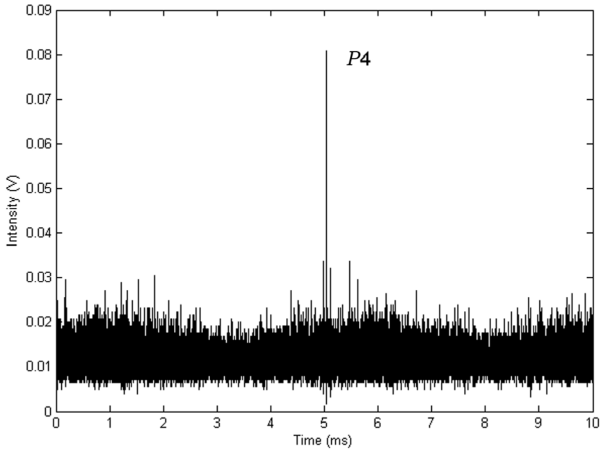

Figure 2. Oscilloscope recording of laser oscillations demonstrating a rogue wav Rare jumps to the period 4 states occur when loss-pass filtered noise with cutting frequency $f_n = 7kHz$ and amplitude $k = 0.5V$ is applied. $F_m = 90kHz,\ m = 0.8V$.
Source: Authors own elaboration.

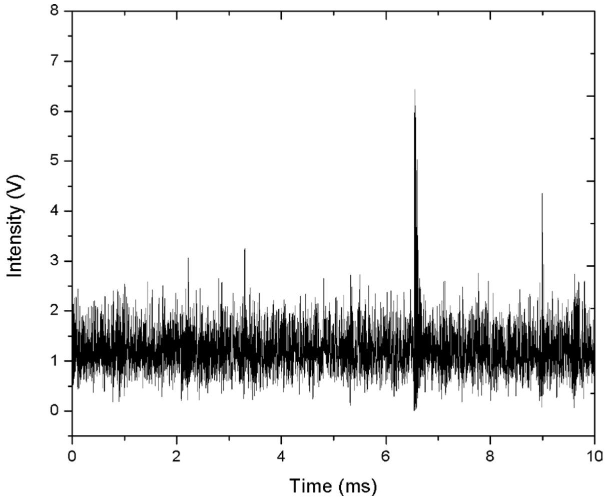

Figure 3. Numerically calculated rogue wave of period 4 ($P4$) when noise with $f_n = 7\ kHz$ and $\kappa = 0.9$ is applied. $f_d = 80\ kHz$, $m = 1$.
Source: Authors own elaboration.

CONCLUSIONS

A new mechanism for rogue wave emergence has been found and experimentally verified in a multistable EDFL, subject to both periodic and slow stochastic modulation applied to the diode pump laser. Rogue wave main properties of the observed laser spikes are confirmed by the wave amplitude PDF.

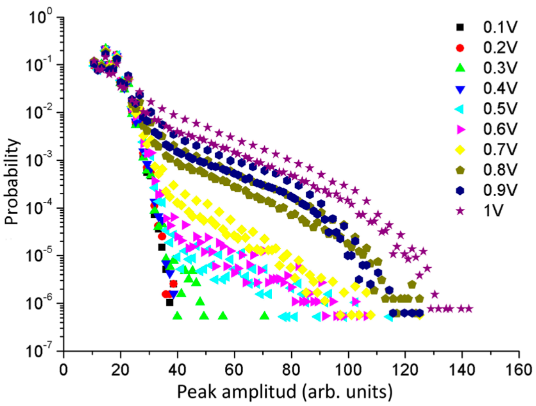

Figure 4. Experimental probability density functions of laser peak intensity for different amplitudes of noise filtered at $f_n = 7\ kHz$. (Pisarchik *et al*., 2011; Pisarchik *et al*., 2005b; Pisarchik & Jaimes-Reátegui, 2009).

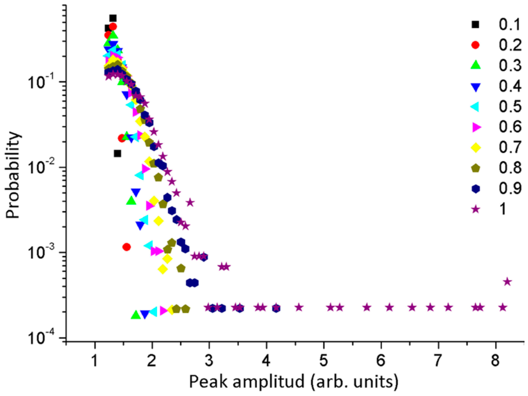

Figure 5. Numerically calculated probability density functions of laser peak intensity for different amplitudes of noise filtered at $f_n = 7\ kHz$. (Pisarchik *et al*., 2011; Pisarchik *et al*., 2005b; Pisarchik & Jaimes-Reátegui, 2009).

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